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① EAS 2024

Dapatkan luas daerah
yang dibatasi oleh Kurva

$$x = y^2 \text{ dan } 2y + x = 3.$$

$$x = 3 - 2y$$

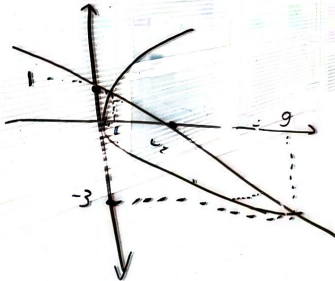
Jawab:

$$10 \quad 2y + x = 3$$

$$x = 0 \rightarrow y = \frac{3}{2}$$

$$y = 0 \rightarrow x = 3$$

$$(0, \frac{3}{2}), (3, 0)$$



$$y_1 = y_2$$

$$\sqrt{x} = \frac{3-x}{2}$$

$$2\sqrt{x} = 3-x$$

$$4x = 9 - 6x + x^2$$

$$x^2 - 10x + 9 = 0$$

$$(x-1)(x-9) = 0$$

$$x = 1 \quad \sqrt{x} = 9$$

10 Luas

$$L = \int_{-3}^1 (3-2y) - y^2 dy$$

$$= 3y - y^2 - \frac{1}{3}y^3 \Big|_{-3}^1$$

$$= 3 - 1 - \frac{1}{3} - (-9 - 9 + 9)$$

$$= 2 - \frac{1}{3} - (-9)$$

$$= 11 - \frac{1}{3}$$

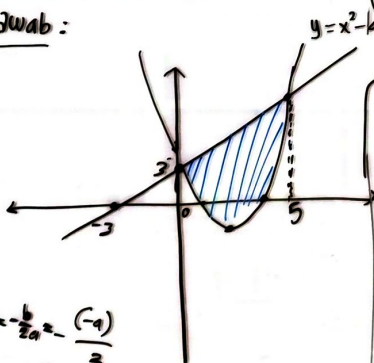
$$= \frac{32}{3} \text{ Satuan luas}$$

② EAS 2024

Kurva: $y = x^2 - 4x + 3$

Luas? $y = x + 3$

Jawab:



$$x_p = -\frac{b}{2a} = \frac{-(-4)}{2}$$

$$y_p = (-2)^2 - 4(-2) + 3$$

$$= -1$$

$$(2, -1)$$

$$y = 0 \rightarrow x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

$$x = 1 \vee x = 3$$

$$(1, 0) (3, 0)$$

$$x = 0 \rightarrow y = 3 (0, 3)$$

• > Titik Potong Kurva

$$y_1 = y_2$$

$$x^2 - 4x + 3 = x + 3$$

$$x^2 - 5x = 0$$

$$x(x-5) = 0$$

$$x = 0 \vee x = 5$$

$$L = \int_0^5 x + 3 - (x^2 - 4x + 3) dx$$

$$= \int_0^5 x + 3 - x^2 + 4x - 3 dx$$

$$= \int_0^5 -x^2 + 5x dx$$

$$= -\frac{1}{3}x^3 + \frac{5}{2}x^2 \Big|_0^5$$

$$= -\frac{125}{3} + \frac{125}{2} - 0$$

$$= \frac{-250 + 375}{6}$$

$$= \frac{125}{6} \text{ Satuan luas}$$

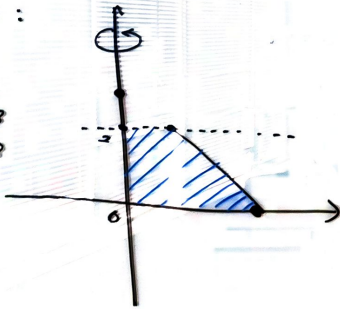
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- ③ Luas permukaan, Kurva:
 $x = 3 - y$, $y = 0$, $y = 2$
 diputar terhadap sumbu $-y$.
 Sketsa kurvanya!

Jawab :

$x = 3 - y$
 $x = 0 \rightarrow y = 3$
 $y = 0 \rightarrow x = 3$



$$\begin{aligned}
 K &= \int_c^d 2\pi g(y) \sqrt{1 + [g'(y)]^2} dy \\
 &= \int_0^2 2\pi (3-y) \sqrt{1 + [g'(y)]^2} dy \\
 * \quad g'(y) &= -1 \\
 [g'(y)]^2 &= 1 \\
 &= 2\pi \int_0^2 (3-y) \sqrt{1+1} dy \\
 &= 2\pi \int_0^2 (3-y) \sqrt{2} dy \\
 &= 2\sqrt{2} \pi \int_0^2 3-y dy \\
 &= 2\sqrt{2} \pi \left[3y - \frac{1}{2} y^2 \right]_0^2
 \end{aligned}$$

$$\begin{aligned}
 &= 2\sqrt{2} \pi [6 - 2 - 0] \\
 &= 2\sqrt{2} \pi [4] \\
 &= 8\sqrt{2} \pi \text{ satuan luas}
 \end{aligned}$$

④ Luas permukaan, Kurva:

$$y = \sqrt{16-x^2}, \quad -3 \leq x \leq 3$$

Diputar thd. Sumbu-x!

Jawab:

$$y = \sqrt{16-x^2}$$

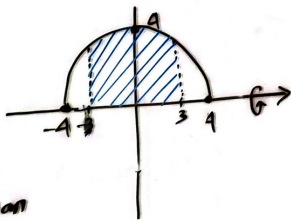
$$y^2 = 16-x^2$$

$$x^2 + y^2 = 16$$

* Pers. Umum lingkaran

$$(x-a)^2 + (y-b)^2 = r^2$$

Pusat: (a,b), jari-jari r



$$K = \int_{-3}^3 2\pi (\sqrt{16-x^2}) \cdot \sqrt{1+[f'(x)]^2} dx$$

* Turunan $f(x)$

$$f(x) = \sqrt{16-x^2}$$

$$= (16-x^2)^{1/2}$$

$$f'(x) = \frac{1}{2} \cdot (16-x^2)^{-1/2} \cdot (-2x)$$

$$= \frac{-x}{\sqrt{16-x^2}}$$

$$\Rightarrow [f'(x)]^2 = \frac{x^2}{16-x^2}$$

$$K = 2\pi \int_{-3}^3 \sqrt{16-x^2} \cdot \sqrt{1 + \frac{x^2}{16-x^2}} dx$$

$$= 2\pi \int_{-3}^3 \sqrt{16-x^2} \sqrt{\frac{16-x^2+x^2}{16-x^2}} dx$$

$$= 2\pi \int_{-3}^3 \sqrt{16-x^2} \frac{\sqrt{16}}{\sqrt{16-x^2}} dx$$

$$= 2\pi \int_{-3}^3 4 dx$$

$$= 8\pi \cdot x \Big|_{-3}^3$$

$$= 8\pi [3 - (-3)]$$

$$= 8\pi \cdot 6$$

$$= 48\pi \text{ Satuan luas}$$

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⑤ EAS 2025

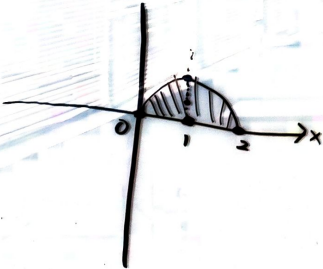
Titik berat kurva lingkaran bagian atas $y = \sqrt{1 - (x-1)^2}$ dan sumbu x

Jawab :

$$y = \sqrt{1 - (x-1)^2}$$

$$y^2 = 1 - (x-1)^2$$

$$(x-1)^2 + y^2 = 1 \rightarrow \text{O, pusat } (1,0), r=1$$



$$\bar{x} = \frac{\int_a^b x [y_1 - y_2] dx}{\int_a^b y_1 - y_2 dx}$$

$$\bar{y} = \frac{\int_a^b \frac{1}{2} [y_1^2 - y_2^2] dx}{\int_a^b y_1 - y_2 dx}$$

► Titik Berat

$$\bar{x} = 1, \text{ karena tuasannya}$$

Simetri thd garis $x=1$

$$L \odot = \pi r^2$$

$$L_{\text{area}} = \frac{1}{2} \cdot L \odot$$

$$= \frac{1}{2} \cdot \pi r^2$$

$$= \frac{1}{2} \pi$$

$$= \frac{1}{2} \pi$$

$$\frac{1}{2} \int_a^b [y_1^2 - y_2^2] dx$$

$$= \frac{1}{2} \int_0^2 [1 - (x-1)^2 - 0] dx$$

$$= \frac{1}{2} \int_0^2 [1 - (x^2 - 2x + 1)] dx$$

$$= \frac{1}{2} \int_0^2 [-x^2 + 2x] dx$$

$$= \frac{1}{2} \left[-\frac{1}{3}x^3 + x^2 \right]_0^2$$

$$= \frac{1}{2} \left[-\frac{8}{3} + 4 \right]$$

$$= \frac{1}{2} \left[\frac{4}{3} \right]$$

$$= \frac{2}{3}$$

$$\bar{y} = \frac{2/3}{\frac{1}{2}\pi} = \frac{2}{3} \cdot \frac{2}{\pi}$$

$$= \frac{4}{3\pi}$$

$$T_{\text{berat}} = \left(1, \frac{4}{3\pi} \right)$$

⑥ EAS 2025

Pers. Parametrik: $x = 3 \cos t + 2$, $y = 3 \sin t$

a. Dapatkan $\frac{dx}{dt}$, $\frac{dy}{dt}$, $\frac{dy}{dx}$

b. Dapatkan pers. Garis Singgung

di $t = \frac{\pi}{3}$

Jawab:

$$\begin{aligned} \text{a). } \frac{dx}{dt} &= -3 \sin t & \frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \rightarrow \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \\ \frac{dy}{dt} &= 3 \cos t & &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \cdot \frac{dt}{dt} \\ & & &= \frac{3 \cos t}{-3 \sin t} \\ & & &= -\cot t \end{aligned}$$

b). Pers. Garis Singgung

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

• Titik Singgung

$$x = 3 \cos t + 2$$

$$t = \frac{\pi}{3} \rightarrow x = 3 \cos\left(\frac{\pi}{3}\right) + 2$$

$$= 3 \cdot \frac{1}{2} + 2$$

$$= \frac{7}{2}$$

$$y = 3 \sin t$$

$$t = \frac{\pi}{3} \rightarrow y = 3 \sin\left(\frac{\pi}{3}\right)$$

$$= 3 \cdot \frac{1}{2} \sqrt{3}$$

$$= \frac{3}{2} \sqrt{3}$$

• Gradien

$$m = \frac{dy}{dx} = -\cot t$$

$$t = \frac{\pi}{3} \rightarrow m = -\cot\left(\frac{\pi}{3}\right)$$

$$= -\frac{\cos\left(\frac{\pi}{3}\right)}{\sin\left(\frac{\pi}{3}\right)}$$

$$= -\frac{\frac{1}{2}}{\frac{1}{2}\sqrt{3}}$$

$$m = -\frac{1}{\sqrt{3}}$$

• Pers. Garis Singgung

$$y - y_1 = m(x - x_1)$$

$$y - \frac{3}{2}\sqrt{3} = -\frac{1}{\sqrt{3}}\left(x - \frac{7}{2}\right)$$

$$y = -\frac{1}{\sqrt{3}}\left(x - \frac{7}{2}\right) + \frac{3}{2}\sqrt{3}$$

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7) EAS 2025

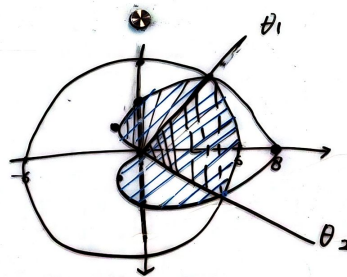
Luas daerah di dalam lingkaran
 $r=6$ dan di dalam Kardioida

$$r = 4 + 4\cos\theta$$

Jawab:

► Grafik

θ	0°	60°	90°	120°	180°	240°	270°
r	8	6	4	2	0	2	4



→ Cari Titik Potong

$$r_1 = r_2$$

$$4 + 4\cos\theta = 6$$

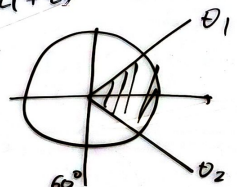
$$4\cos\theta = 2$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60^\circ, 300^\circ$$

$$L_{\text{tot}} = L_1 + L_2$$

$$* L_1 =$$



$$L_1 = \int_{-60^\circ}^{60^\circ} \frac{1}{2} \cdot (6)^2 d\theta$$

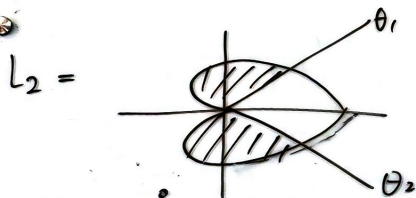
$$= 18 \int_{-60^\circ}^{60^\circ} d\theta$$

$$= 18 \cdot \theta \Big|_{-60^\circ}^{60^\circ}$$

$$= 18 \left[\frac{\pi}{3} - \left(-\frac{\pi}{3}\right) \right]$$

$$= 18 \cdot \frac{2\pi}{3}$$

$$= 12\pi \text{ Satuan Luas}$$



$$= \int_{60^\circ}^{300^\circ} \frac{1}{2} \cdot (4 + 4 \cos \theta)^2 d\theta$$

$$= \frac{1}{2} \int_{60^\circ}^{300^\circ} 16 + 32 \cos \theta + 16 \cos^2 \theta d\theta$$

$$\cos^2 \theta = \frac{1}{2} (1 + \cos 2\theta)$$

$$\sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta)$$

$$= \int_{60^\circ}^{300^\circ} 8 + 16 \cos \theta + 8 \left[\frac{1}{2} (1 + \cos 2\theta) \right] d\theta$$

$$= \int_{60^\circ}^{300^\circ} 8 + 16 \cos \theta + 4(1 + \cos 2\theta) d\theta$$

$$= \int_{60^\circ}^{300^\circ} 12 + 16 \cos \theta + 4 \cos 2\theta d\theta$$

$$= \left(12\theta + 16 \sin \theta + \frac{4}{2} \sin 2\theta \right) \Big|_{60^\circ}^{300^\circ}$$

$$= 12 \left(\frac{5\pi}{3} \right) + 16 \cdot \left(-\frac{1}{2} \sqrt{3} \right) + 2 \cdot \left(-\frac{1}{2} \sqrt{3} \right)$$

$$- \left[12 \cdot \frac{\pi}{3} + 16 \cdot \frac{1}{2} \sqrt{3} + 2 \cdot \frac{1}{2} \sqrt{3} \right]$$

$$= 20\pi - 4\pi - 8\sqrt{3} - \sqrt{3} - 8\sqrt{3} - \sqrt{3}$$

$$= 16\pi - 18\sqrt{3} \text{ Satuan Luas}$$

$$L_{\text{tot}} = 12\pi + 16\pi - 18\sqrt{3}$$

$$= 28\pi - 18\sqrt{3} \text{ Satuan Luas}$$

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8) EAS 2025

Tentukan :

- 5 suku pertama
- Konvergensi
- Apabila konvergen, tentukan nilainya

$$\text{Barisan : } \left\{ \frac{n}{3n+1} \right\}_{n=1}^{+\infty}$$

Jawab: a) $a_1 = \frac{1}{4}$ $a_3 = \frac{3}{10}$ $a_5 = \frac{5}{16}$
 $a_2 = \frac{2}{7}$ $a_4 = \frac{4}{13}$

$$b). \lim_{n \rightarrow +\infty} \frac{n}{3n+1} = \frac{\infty}{\infty} \text{ (tak tentu)}$$

$$\lim_{n \rightarrow +\infty} \frac{n}{3n+1} \cdot \frac{1/n}{1/n}$$

$$= \lim_{n \rightarrow +\infty} \frac{1}{3 + 1/n}$$

$$= \frac{1}{3+0}$$

$$= \frac{1}{3} \text{ (konvergen)}$$

c). Konvergen ke $\frac{1}{3}$